



Northeastern

Healthcare Reliability Science

Deliberate Design

James C. Benneyan, PhD, Director

Healthcare Systems Engineering Institute

CMS Innovation Healthcare Systems Engineering Center

NSF Center for Health Organization Transformation

Northeastern University, Boston MA

www.HSyE.org



Boston HSyE Extension



Systems Engineering domains

3 Core Areas

	Area	Focus	Typical methods
1	System/process design	Design (re-design) <i>new</i> system to meet certain functional criteria	Systems engineering Design for X Business process re-engineer
2	System/process improvement	Improve performance of <i>existing</i> system	Quality improvement Lean, six sigma, PDSA Statistical analysis Model-based improvement
3	System/process optimization	Optimize performance <i>existing or new</i> system	Mathematical optimization Computer simulation Machine learning Analytics

X = Manufacturing, dis-assembly, agility, reliability, quality, robustness, etc



Design for Reliability

Outline / Learning objectives

1. Context and definitions
2. 'Reliability science' (deliberate design)
3. Evidence-based bundles
4. Actual reliability engineering
5. Discussion



1.

context and definitions

Types & definitions of reliability

Application Area	Focus, definition
Classic engineering-industry reliability	<p>“Ability of a system or component to perform its required functions under stated conditions for a specified period of time”</p> <p><i>Time until failure of product or system.</i></p> <p>$R(t) = P(T > t)$, MTTF, other statistical definitions</p>
Measurement and instrument analysis (metrology)	Instrument and measurement process <i>invariance</i> (precision & accuracy, repeatability & reproducibility)
Assessment scale or survey reliability	<i>Consistency</i> among independent measurement methods, including temporal stability, form equivalence, and internal consistency. Kappa, etc
Healthcare process reliability	Time until a non-compliant (defect) event. Reliability = <i>conformance rate</i> . (how often get it right, NCPPM)



Simplified Definition of Reliability

“Failure free performance over time”

- ◆ Focus: Time between failures (longer is better)
- ◆ (Really a blending of reliability, quality, safety, and process capability concepts.)
 - Adapting concepts from study of industry, safety, and highly reliable systems
- ◆ Proving to be a useful framework in healthcare



“10-X” language

Decimal
location

- | | | |
|---------------------------|---|--------------|
| “10⁻¹”: | Single digit failures per 10 opportunities
0.1 to 0.9 failure rate
(technically .1) | 0.x |
| “10⁻²”: | Single digit failures per 100 opportunities
0.01 to 0.09 failure rate
(technically .01) | 0.0x |
| “10⁻³”: | Single digit failures per 1000 opportunities
0.001 to 0.009 failure rate
(technically .001) | 0.00x |
| etc... | | |



Reliability Levels

Specification - What reliability do you want?

Same basic concepts and how terminology relates

Reliability	Failure Rate	10^{-x}	PPM (parts per million)	Sigma's
.9	.1	10^{-1}	100,000	1.64
.99	.01	10^{-2}	10,000	2.58
.999	.001	10^{-3}	1,000	3.29
.9999	.0001	10^{-4}	100	3.89
.99999	.00001	10^{-5}	10	4.42
.999999	.000001	10^{-6}	1	4.89

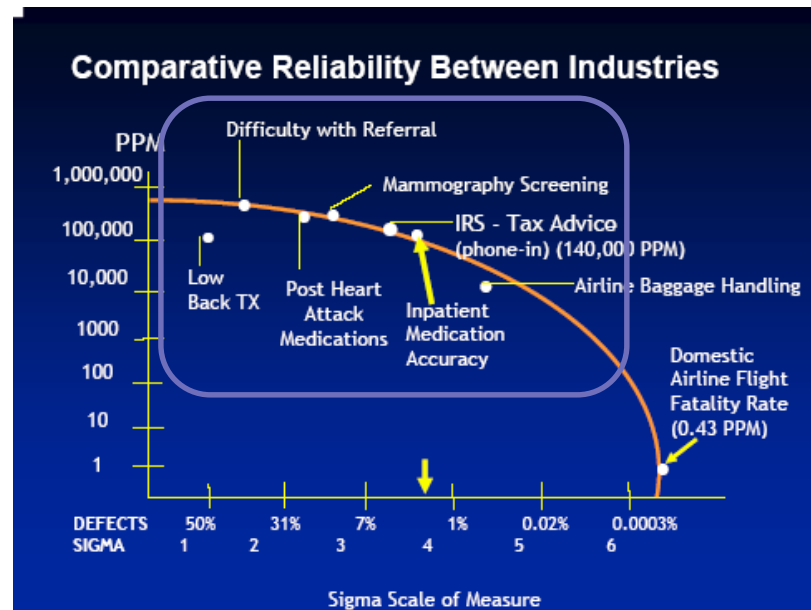


Time to failure in other industries

Industry	Application / Component	MTTF (hours of use)
Aircraft engines	Failure of turbine blade (crack exceeding ¼ inch length)	10,000 hrs
Sleeve bearings	Mechanical wear of bearing in a cooling fan due to friction	30,000 hrs
Computer chips	Time until chip fails	15,500 hrs
Power supply	Power inverters	156,585 hrs
	Switches	417,560 hrs
	Chargers	626,340 hrs
Telecommunications	Ethernet switches	1,053,439 hrs
	Modems	515,424 hrs

Poor Reliability at all Sigma Levels

Failure Rate	Outcome or Process Measure
10^{-1}	EBM-CMS conditions (e.g., beta blockers for MI) Adverse events Hospital readmissions
10^{-2}	Deaths in risky surgery Polypharmacy in the elderly
10^{-3}	Ventilator associated pneumonia (2-8 VAP/1000 vent. days) Adverse drug events (1-6 ADE's / 1000 doses)
10^{-4}	Deaths in routine anesthesia
10^{-5}	Deaths from major radiotherapy machine failures
10^{-6}	Deaths from seismic non-compliance



Reliability of Patient Safety

Estimates (IOM 1999, etc)

Medical errors & iatrogenic injury:

- 98,000 deaths/year
- 770,000 - 2 million patient injuries
- \$17 - \$29 billion dollars

Adverse drug events (ADE):

- 770,000 to 2 million per year
- \$4.2 billion annually

Hospital-acquired infections:

- 2 - 5 million NSI/year, \$3,000/case
- 8.7 million hospital days
- 20,000 deaths/year

More US deaths/year than for traffic accidents, breast cancer, & AIDS.

Endemic AE's

6 - 10% hospital patients suffer ≥ 1 serious adverse events

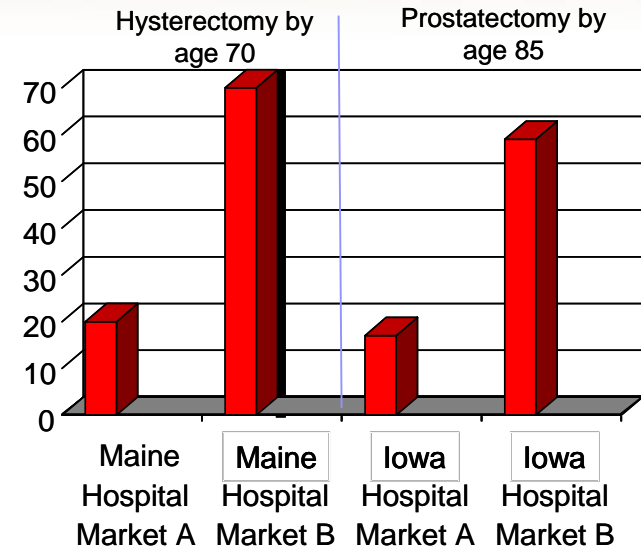
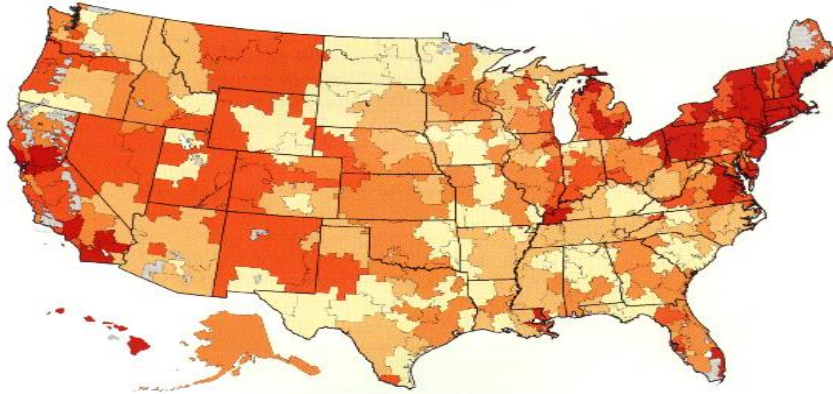
- Adverse drug events (ADE)
- Surgical site infections (SSI)
- Needle sticks
- Wrong side/site surgery
- Device-associated infections
 - Ventilator-associated pneumonia
 - Catheter & central line infections

Per episode average costs:

- ADE: \$4,000 - \$5,000
- NSI: \$2,000 - \$3,000
- VAP: 13 additional days & 30 - 50% attributable mortality
- SSI: Can exceed \$14,000

2006 study: 195,000 deaths, \$6 billion/year

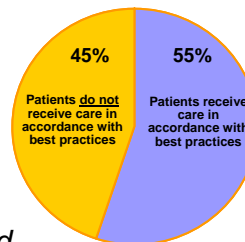
Practice reliability



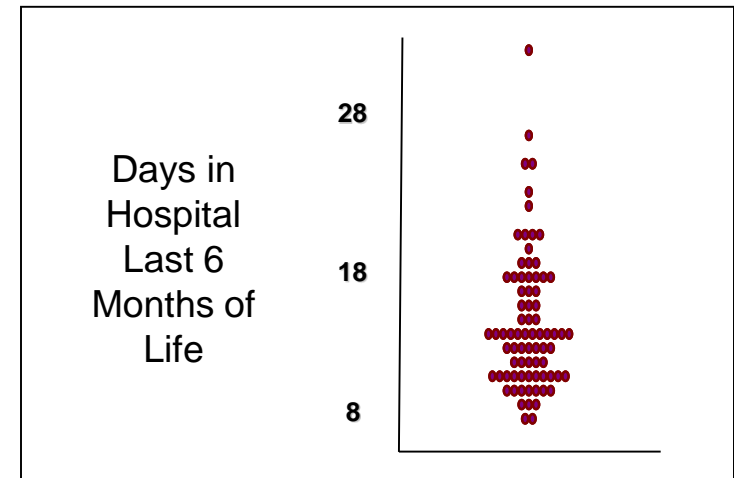
Compliance to Evidence Base

Estimated only ~55% of U.S. patients receive scientifically indicated care:

- Acute: 54%
- Chronic: 56%
- Preventive: 55%



McGlynn, et al: *The quality of health care delivered to adults in the United States*. NEJM 2003; 348:2635-2645



Wennberg, 2005, From study of patients with severe chronic illness who received most of their care in one of 77 "best" U.S. hospitals

'Bundle' example

The Science Journal of the American Association for Respiratory Care

COMPLIANCE WITH THE VAP BUNDLE ACROSS MULTIPLE INTENSIVE CARE UNITS

R. Wolff¹, K. D. Hargett¹, B. Green¹

Zone	Head of Bed	Stress DVT prophylaxis	Stress Ulcer prophylaxis	Daily Sedation Holiday	Readiness to Extubate	Total Bundle Compliance
SICU	99.4%	99.4%	99.4%	68.6%	53.8%	55.2%
CCU	83.8%	99.1%	100.0%	30.0%	61.5%	65.7%
CVICU	88.5%	98.6%	100.0%	64.6%	64.6%	65.6%
MICU	99.1%	99.5%	99.5%	85.6%	90.5%	83.1%
NICU	99.1%	98.3%	98.3%	56.7%	53.4%	61.5%
Hospital	94.0%	99.0%	99.4%	60.3%	64.8%	66.2%

Many other examples...

Other bundle examples

AMI	CHF	CAP	CABG	TJ	VAP
ASA (aspirin) within 24 hrs	LVEF assessed w_1	O ₂ assessed within 24 hours	First dose antibiotic 1 hour prior to first incision	First dose antibiotic 1 hour prior to first incision	Head of bed elevation
BETA blocker within 24 hrs	ACE Inhibitor for LVSD w_2	Blood culture prior to abx	Appropriate antibiotic selection	Appropriate antibiotic selection	DVT prophylaxis
Thrombolytics within 30 min	Detailed discharge instructions w_3	First dose of antibiotic within 4 hours	Discontinued antibiotics within 24 hours	Discontinued antibiotics within 24 hours	Sedation vacation
PCI within 120 minutes	Smoking cessation counseling w_4	Influenza screening / vaccination	Use internal mammary artery (IMA)		PUD prophylaxis
ACE Inhibitor for LVSD		Pneumococ. vaccination	ASA at discharge		4 care elements
Smoking counseling					
ASA at d/c					
BETA blocker at discharge					

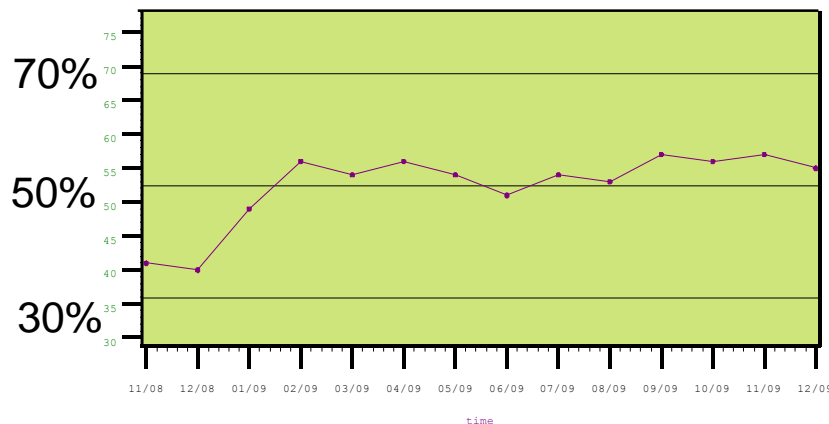
2. Center for Medicare & Medicaid Services (CMS), "Clinical conditions and measures for reporting and incentives" 2003.
Others: AHA, Leapfrog Group, National Quality Forum, AHQR, JCAHO Core Measures

Reliability in doing what is known

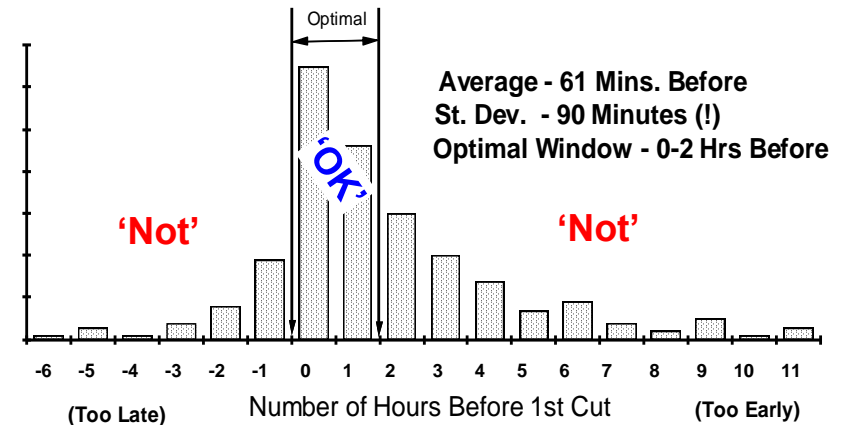
Recommended Care Received $\approx \frac{1}{2}$ ¹

64.7%	Hypertension
63.9%	Congestive Heart Failure
53.9%	Colorectal cancer
53.5%	Asthma
45.4%	Diabetes
39.0%	Pneumonia
22.8%	Hip Fracture

Hand washing compliance rate



SSI Antibiotic Timing Example



Patient non-compliance

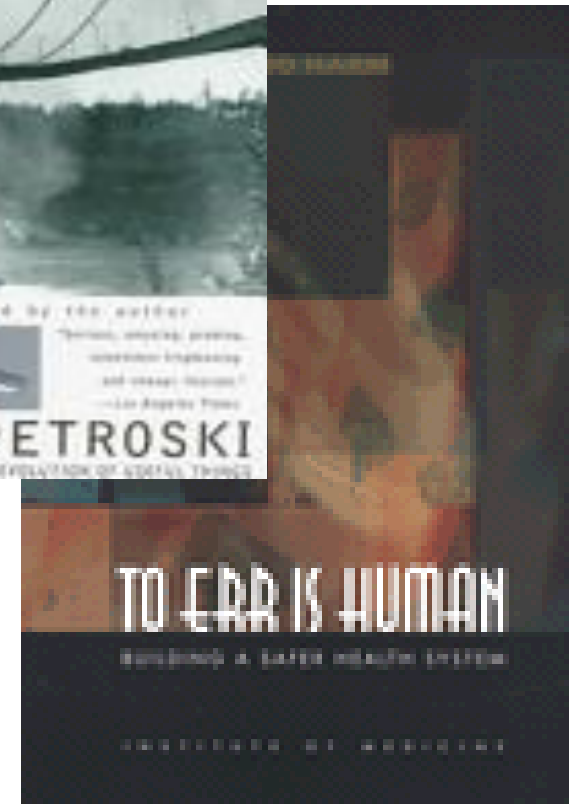
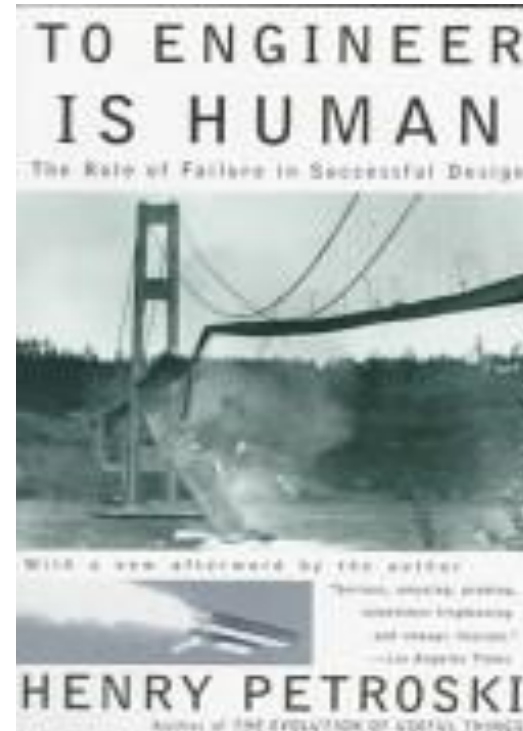
- No better: 36%² – 50%^{3,4}
- Discharge instructions, exercise, meds, diet, (eg, diabetes mgmt)
- ~ \$50 B drug hospitalizations
- ~ 40% nursing home admissions
- ~ \$2,000/patient/yr office visits
- **Total \approx \$177 billion/year²**

1. McGlynn et al; 2. Chan DC et al ; 3. World Health Organization; 4. National Council on Patient Information and Education

2. why not better

Desire vs. Design

- Will and good intentions (desire) alone insufficient
- Good well-meaning people are unreliable
- Human performance is fact of life
- Deliberate design, that exploits rather than ignores human factor



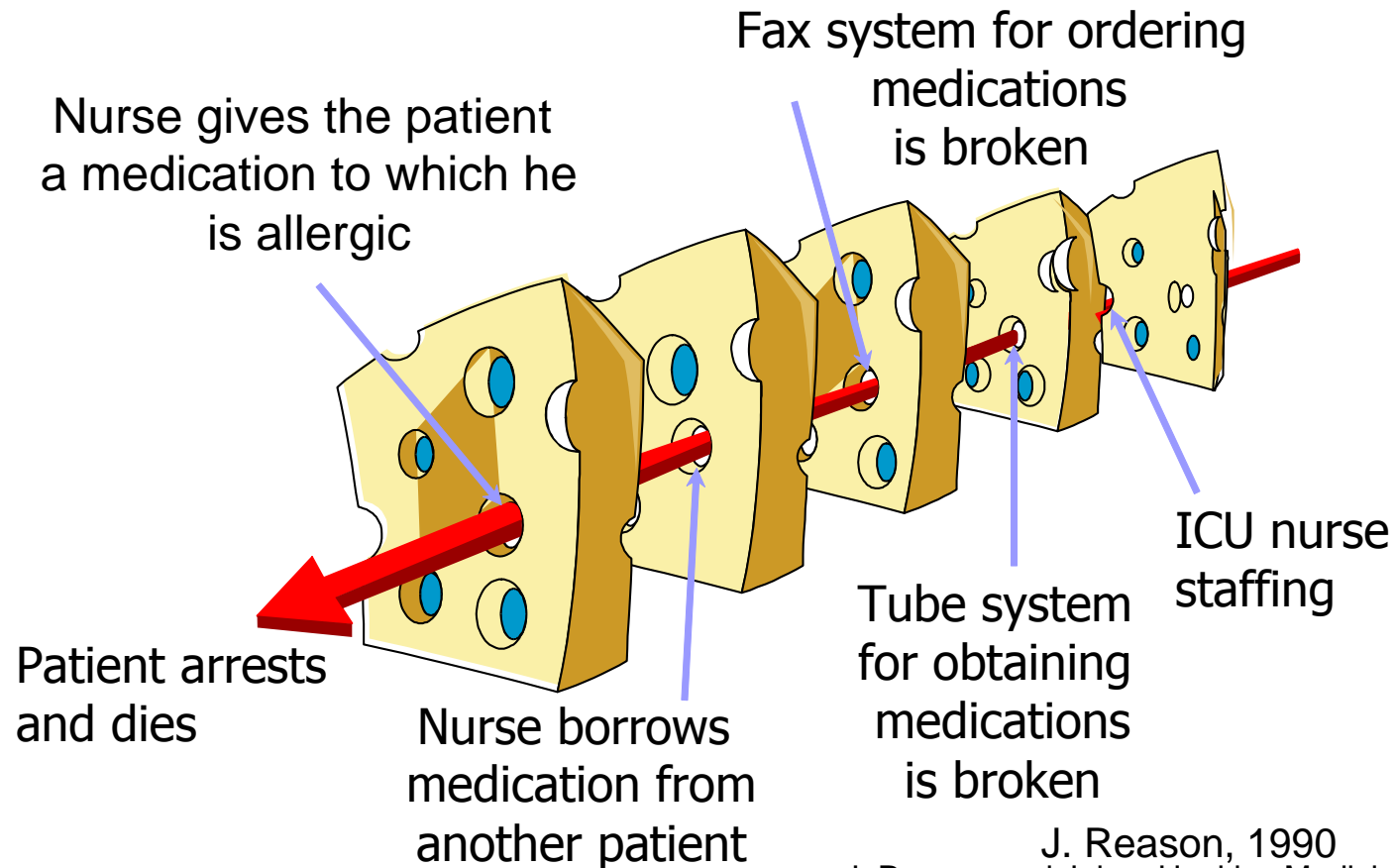
Good intentions, Bad designs

“To Err /S Human”



'Swiss Cheese' Model

Medication Errors



J. Reason, 1990
J. Reason and Johns Hopkins Medicine

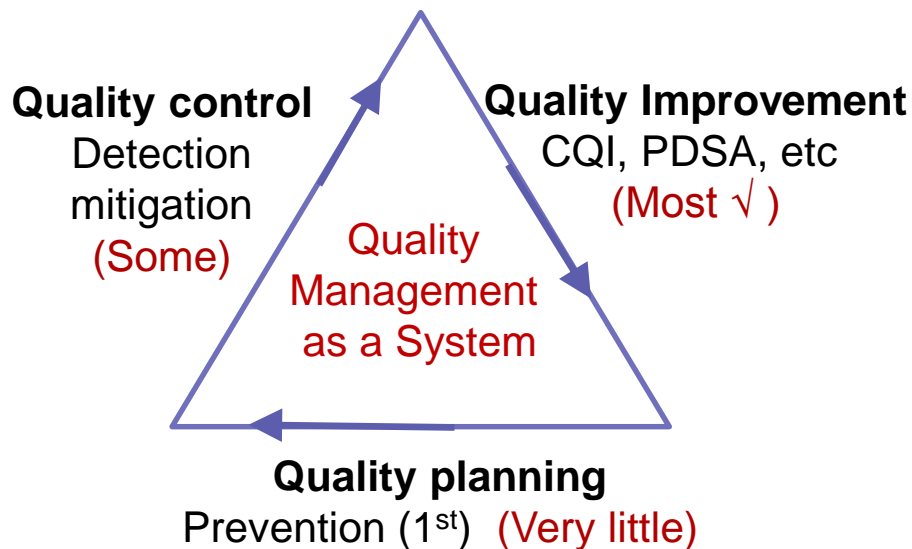
3.

reliability by design

Similar ideas



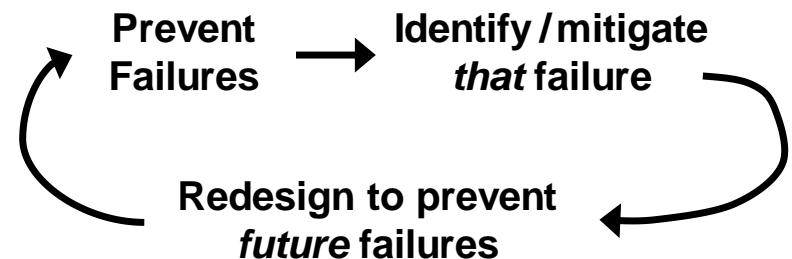
Quality Trilogy (Juran)



Tend to be better at fixing things gone wrong than preventing in 1st place

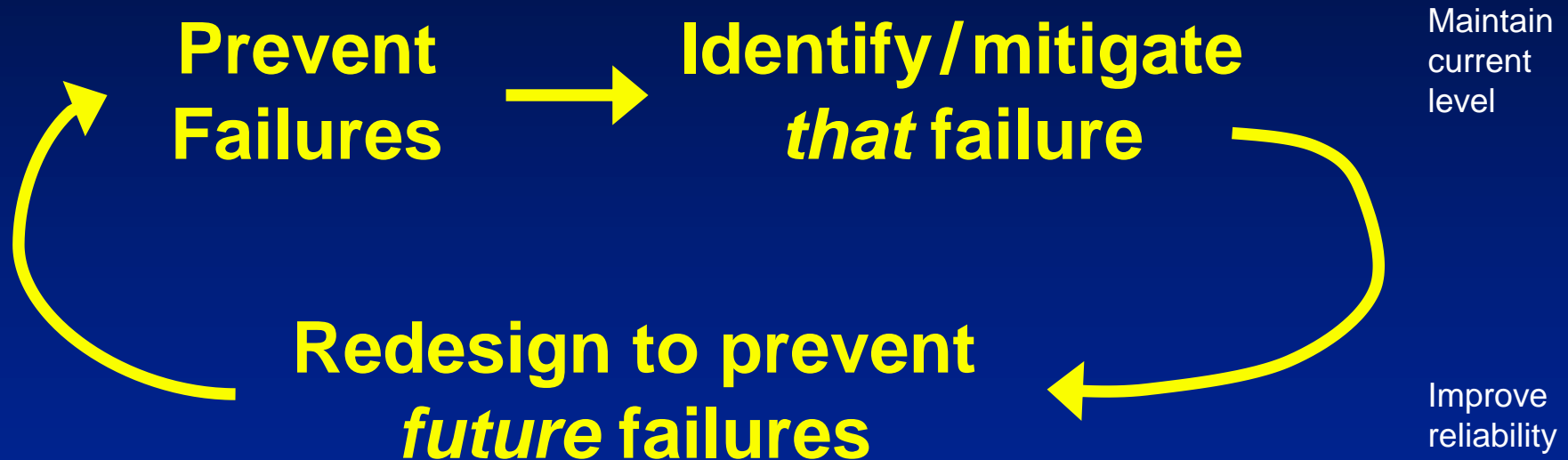
Reliability Model (IHI)

1. **Prevention** strategies
2. **Identify, mitigate** & minimize impact, recovery (QC, other methods)
3. Learn from failures, **improve system** (root cause & failure modes analysis)





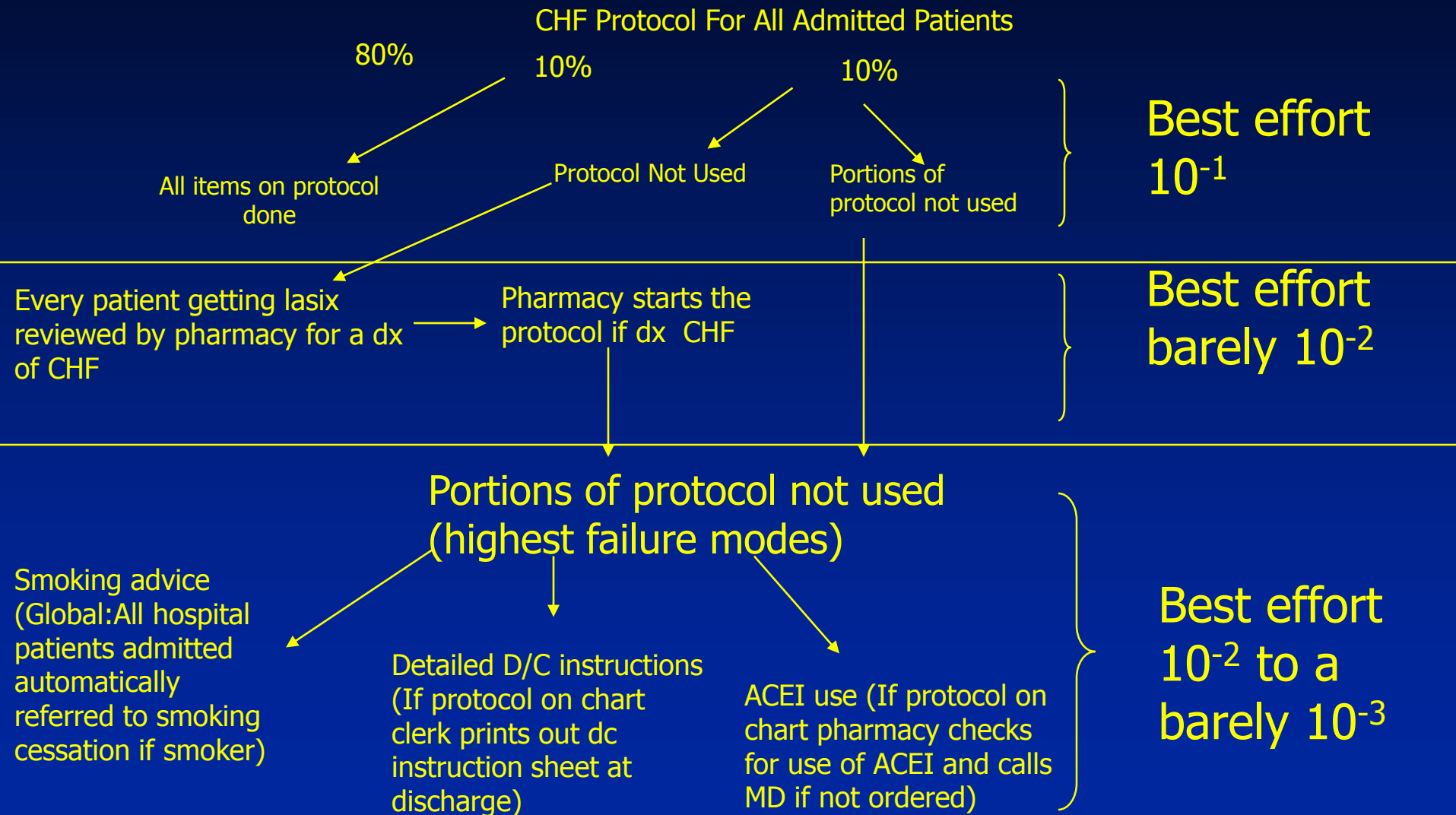
3-Tier Reliability Design Model





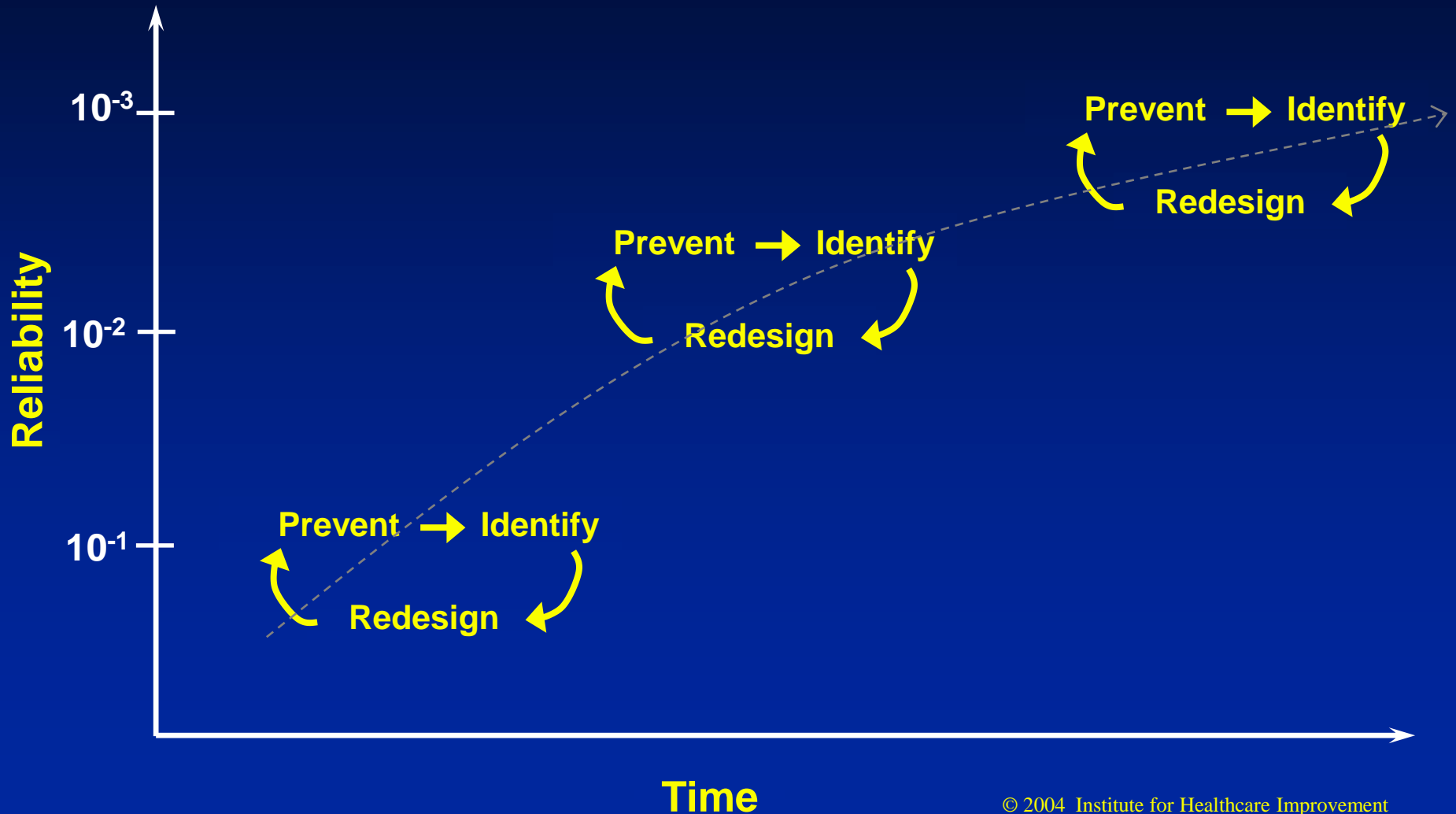
CHF Example

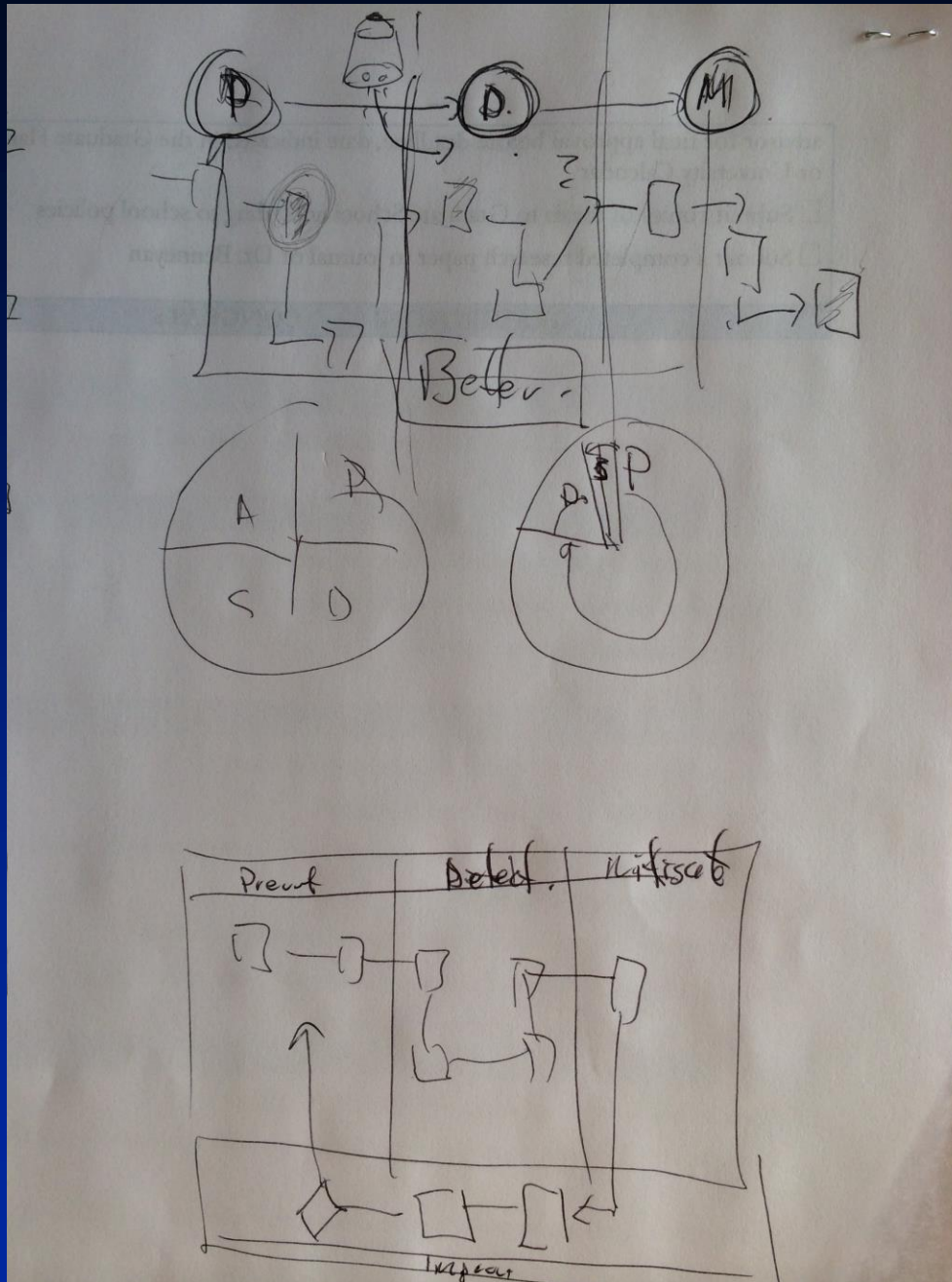
Prevent, Identify, Mitigate





Reliability Growth Over Time





‘Reliability design/science’ model

Reliability tier	Strategies	Measures	
		Process	Outcome
Prevent	<ul style="list-style-type: none">••	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•
Detect	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•
Mitigate	<ul style="list-style-type: none">••	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•
Redesign	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•	<ul style="list-style-type: none">•

Example: Patient falls prevention

Reliability tier	Solution Strategies	Measures	
		Process	Outcome
Prevent (surveillance)	<ul style="list-style-type: none"> Checklist Patient/family engagement 	<ul style="list-style-type: none"> Use of tool on rounds 	<ul style="list-style-type: none"> Percent complete / completed checklists
Detect (assess risk)	<ul style="list-style-type: none"> Assessment tool 	<ul style="list-style-type: none"> Usability of tool? 	<ul style="list-style-type: none"> Use of tool
Mitigate (mitigate risk)	<ul style="list-style-type: none"> Intervention assignment per protocol to at-risk patients (Falls Prevention Toolkit?) 	<ul style="list-style-type: none"> Development and usability of intervention strategies 	<ul style="list-style-type: none"> Following/ compliance to intervention process / protocol
Redesign (refinement)	<ul style="list-style-type: none"> MySafeCare Gap analysis, RCA, and redesign re: incomplete check lists etc 	<ul style="list-style-type: none"> # patient suggestions via MySafeCare Number of RCAs conduction 	<ul style="list-style-type: none"> Number of process improvements

MGH CLABSI Example

Reliability tier	Solution Strategies	Measures	
		Process	Outcome
Prevent (surveillance)	<ul style="list-style-type: none"> Follow bundle by correct insertion, maintenance, access, and removal compliance 	<ul style="list-style-type: none"> % compliance to bundle 	<ul style="list-style-type: none"> Reduce infections & resulting costs Reduce mortalities
Detect (assess risk)	<ul style="list-style-type: none"> Audits on all process measures Infection Control data Teamwork survey 	<ul style="list-style-type: none"> Survey data, # of audits done 	<ul style="list-style-type: none"> Identify process shifts
Mitigate (mitigate risk)	<ul style="list-style-type: none"> Educate new staff about CLABSI Performance updates (flip board, emails, etc.) 	<ul style="list-style-type: none"> % of staff educated # of updates / month 	<ul style="list-style-type: none"> Improve care
Redesign (refinement)	<ul style="list-style-type: none"> Residents discuss line removal on rounds Nurse and resident orientation addresses prevention techniques 	<ul style="list-style-type: none"> % of removals discussed % of lines taken out earlier 	<ul style="list-style-type: none"> Number of process improvements

Reliability design strategies

Defect Level	Strategy		
	Prevent	Identify	Redesign
10 ⁻¹	Education Working harder Feedback Checklists, order sets		
10 ⁻²	Standardization Desired = default Opt-out vs opt-in Reminders Decision aids	Redundancy Automatic checks Others?	Some FMEA & RCA (but often ad hoc)
10 ⁻³	Forcing functions Less human reliance Hardwired processes	others?	More formal FMEA Formal redesign processes

— Cumulative design principles —



Design Principle 1: Prevention / 10^{-1} Concepts



Basic Strategies: Intent, Vigilance & Hard Work

- ◆ Common equipment, standard orders sheets
- ◆ Personal check lists
- ◆ Working harder next time
- ◆ Feedback of information on compliance
- ◆ Awareness and training

NECESSARYbut not SUFFICIENT



EXAMPLE: CHF Measures

- ◆ **Left ventricular function (LVF) assessment**
- ◆ **Detailed discharge instructions**
- ◆ **ACE inhibitor for LVSD**
- ◆ **Smoking cessation advice/counseling**



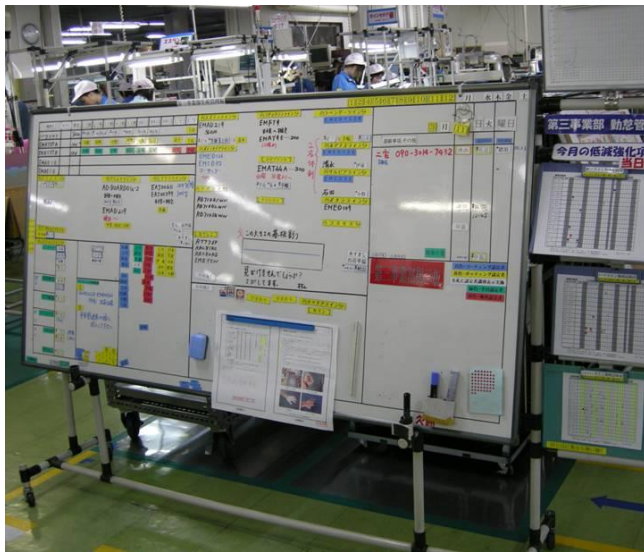
Human Factors Strategies

- ◆ Decision aids and reminders built into the system
- ◆ Desired action the default (based on evidence)
- ◆ Redundancy
- ◆ Scheduling
- ◆ Takes advantage of habits and patterns
- ◆ Standardization of process

ESSENTIAL for movement to 10-2

Human Factors Concepts

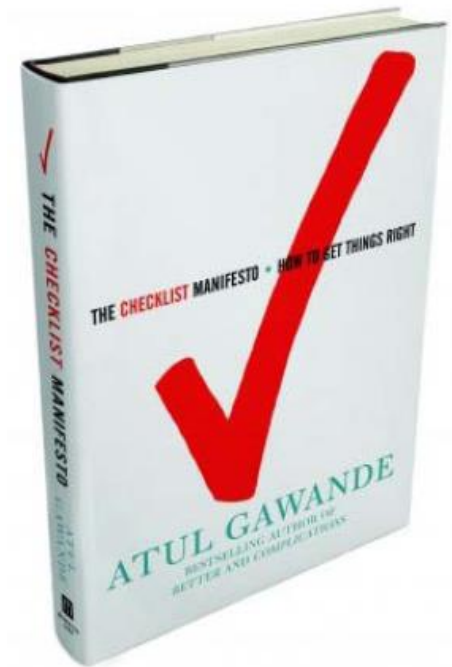
Visual controls



Reminders



Check lists





Design Principle 2: Identification / 10^{-2} concepts



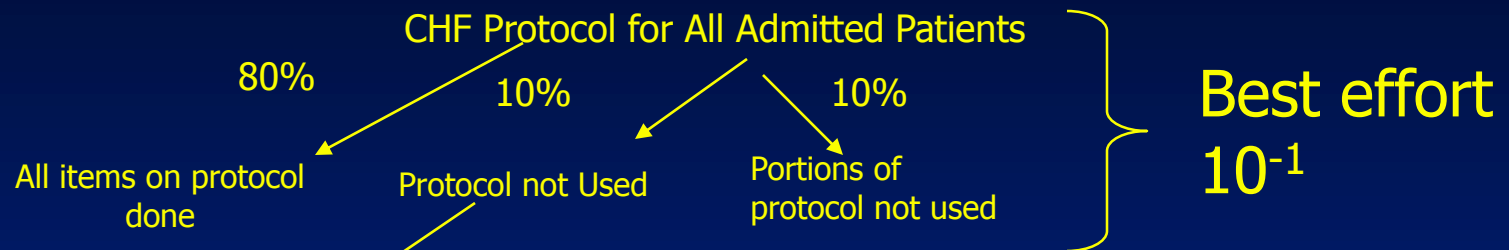
Redundancy Function

- ◆ **Develop a strategy to identify prevention defects**
- ◆ **Develop a strategy to mitigate the defects identified**
- ◆ **Develop a metric to measure standardization failure**



CHF Reliability

Add Identify and Mitigate



Every patient getting major diuretic reviewed by case manager for a dx of CHF

Case Manager puts the protocol on the chart

Best effort barely 10^{-2}

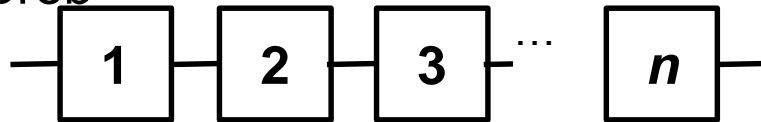
4. reliability strategies


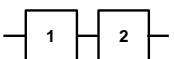
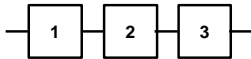
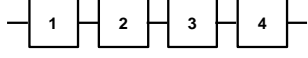
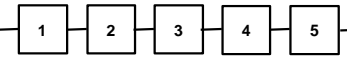
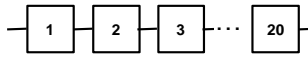
Simplification and redundancy strategies

Fewer steps

Process simplification (lean)

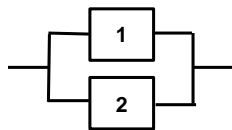
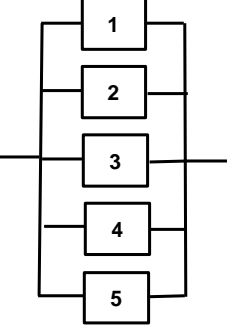
Suppose each step 90% failure prob



n	Overall	
1	$.90^1 \approx 90\%$	
2	$.90^2 \approx 81\%$	
3	$.90^3 \approx 73\%$	
4	$.90^4 \approx 66\%$	
5	$.90^5 \approx 59\%$	
10	$.90^{10} \approx 35\%$	
20	$.90^{20} \approx 12\%$	

More redundancy

Suppose each step 40% failure prob

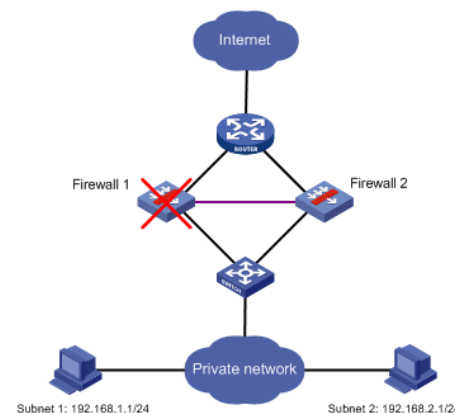
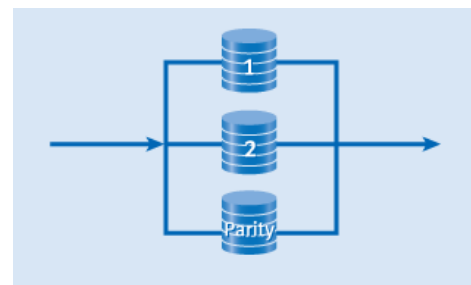
n	Overall	
1	$1 - .40^1 \approx 60\%$	
2	$1 - .40^2 \approx 84\%$	
3	$1 - .40^3 \approx 94\%$	
4	$1 - .40^4 \approx 97\%$	
5	$1 - .40^5 \approx 99\%$	
6	$1 - .40^6 \approx 99.6\%$	
8	$1 - .40^8 \approx 99.9\%$	
10	$1 - .40^{10} \approx 99.99\%$	
15	$1 - .40^{15} \approx 99.9999\%$	
18	$1 - .40^{18} \approx 99.99999\%$	

Redundancy strategy examples

- Car brake system, airplane engines, pilots
 - Computer back-up and RAID systems
 - Extra bridge bolts and supports
-
- Multiple readings of lab results
 - Double checking prescription order
 - Sponge counts
 - Others....?

Industry

Health care



Forcing Function strategies

Definition:

- Design feature completely eliminates failure possibility
- Forces desired outcome to occur automatically and absolutely

Notes:

- 'Poke Yoke' Toyota system
- **Forcing scale – some more absolute than others!**

Examples

- ATM card return
- Car door locks (some)
- Transmission clutch
- Endoscope carrying case latches
- Soap dispensers to open doors
- Airplane “occupied” sign (locks door, turns on light)

Example

One-sigma compliance with “vacant” sign on bathroom door

Current System



Visual reminder



Redundancy



Low level process design ideas ineffective

Compare to airplane forcing function (lock, lights)

From Berwick “Escape Fire” 1999 plenary presentation, Commonwealth Fund website

Systems example

Leaky bucket example

- I can't pump diesel into my car, but station can put diesel in their gas pumps!
- ✓ Reminders
- ✓ Training, Awareness
- ✓ Redundancy
- ⑧ Human fatigue
- ⑧ Poor lighting at night
- ⑧ Sense of urgency

Gas mix-up at Manchester station causes problems for cars

Abenaki Carriers mixed up unleaded, diesel at Manchester Mart and Gas





Design Principle 3: Critical Failure Modes (Redesign)



Critical Failure Mode Function

- ◆ **Develop a process to measure the most common failure modes**
- ◆ **Remodel the process based on the failure modes identified**
- ◆ **Iterative process over time**

FMEA example

- Failure modes effects analysis
- $RPN = \text{Risk priority number} = \text{Severity} \times \text{Occur} \times \text{Detect}$

Function/ Requirement	Potential Failure Mode	Potential Effect of Failure	SEV	Current Process Controls					
				Potential Cause	Occur	Prevention	Detection	Det	R.P.N.
Routing	Patient contracts virus/disease	Illness/Death	10	Unsanitary Conditions	8	None	None	1	80
	ESA/patient get lost and patient diagnosis is delayed	Increase stress and patient leaves	4	Unstandardized Routes	2	Training	None	5	40
				Congestion in Hallways	3	Reopening hallways	Visual	9	108
				Not enough signs/help	8	None	None	5	160
Layout	Delay in Care	Decline in patient health	7	Congestion areas	8	None	Visual	1	56
				Lack of Visibility	8	Nurse button	Visual	9	504
				Clutter	10	None	Visual	9	630
				Non optimal layout	10	None	None	10	700
Billing	Forms not filled out correctly	Unable to code, loss of money	6	No MD signature	6	Business Analyst	Visual	3	108
				No dictation	8	Business Analyst	Visual	3	144
				Low "Level of Care"	10	None	None	5	300
				No Progress Notes	8	Coding Dept.	Visual	7	336

5. actual reliability engineering (and research)

Healthcare reliability (engineering)?

Is this really reliability
(engineering)?

- **No, not really**
- **Blending of quality, safety, process capability**

Litmus tests:

- System stays in failed state until repaired or acted upon
- Can apply to just one item
- Not independent dichotomous failures

What would healthcare
reliability engineering look
like?

- Bath tub curve models
- Mean time between failures (MTTF)
- Availability models
- Maintenance models
- Degradation models
- Fault trees, block diagrams
- others

VAP measure	Comply rate
Head of bed elevation	61%
DVT prophylaxis	75%
Sedation protocol	43%
PUD prophylaxis	88%

[illegible]

Exact probability distribution

Let: $T_W = w_1X_1 + w_2X_2 + \dots + w_JX_J$ and $F_W = T_W/N$, where $X_j \sim \text{bin}(n_j, p_j)$ and w_i 's unbounded

① Then:

Means, Variances, Probability generating functions

②

Probability distribution (convolution)

③

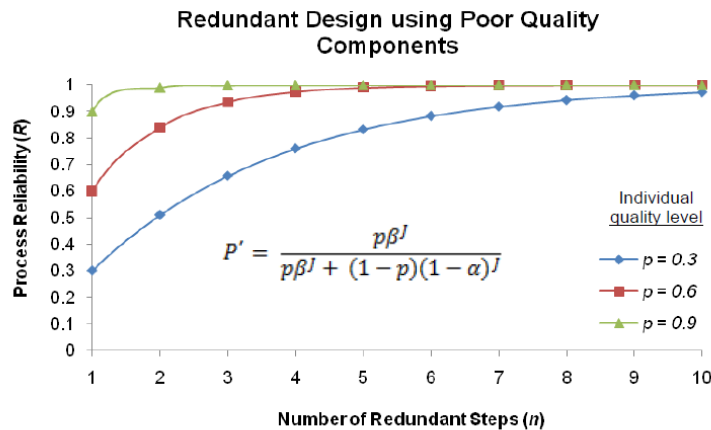
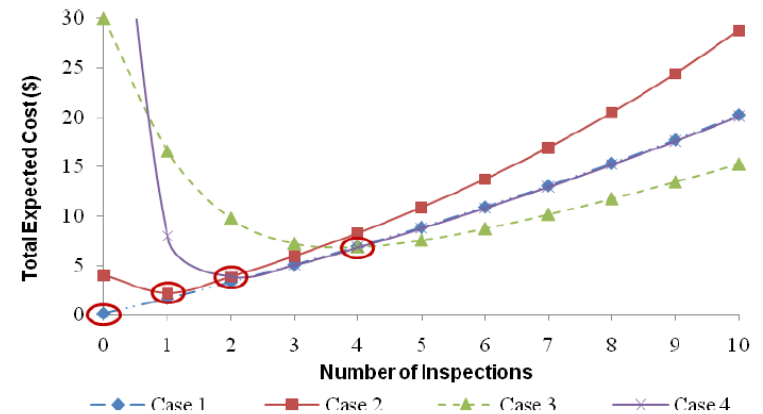
Naïve approximations...?

2. Optimal amount of redundancy

Optimal amount of re-inspection in re-useable medical equipment to prevent cross-contamination (in press)

Governing equations:

$$E(C) = \frac{k_1 \left[\frac{p\alpha(1-\beta)^J + (1-p)(1-\beta)[1-(1-\alpha)^J]}{\alpha(1-\beta)} \right] + k_2 p \beta^J + k_3 (1-p)[1-(1-\alpha)^J]}{(1-p)(1-\alpha)^J + p \beta^J},$$



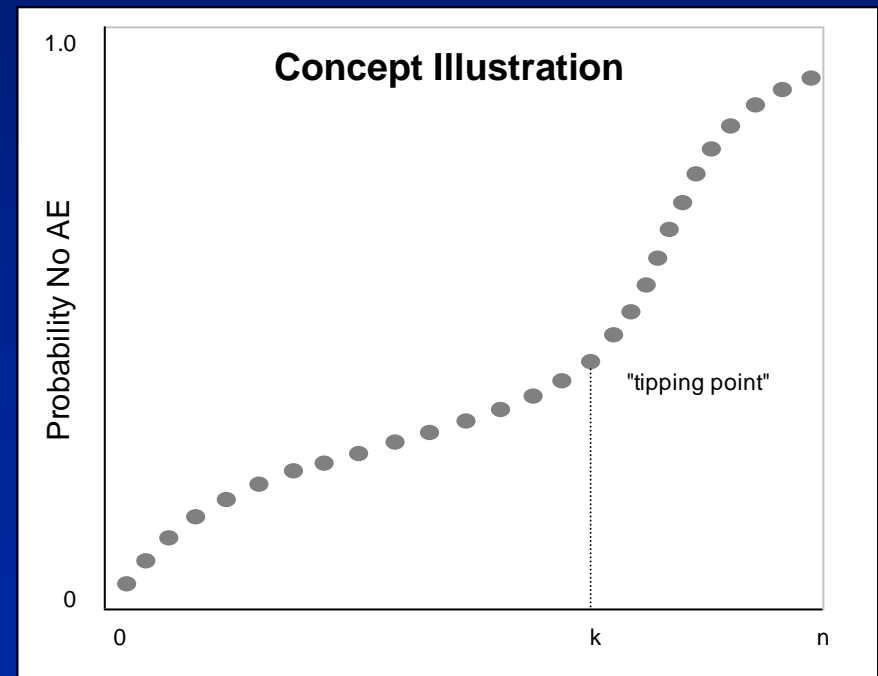
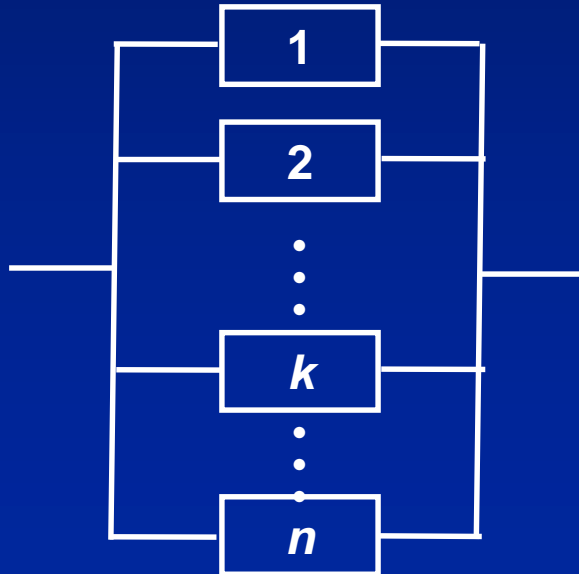
Parameter	Meaning	Case 1	Case 2	Case 3	Case 4
k_1	Inspection cost	\$1	\$1	\$1	\$1
k_2	Failure cost	\$3	\$20	\$100	\$6000
k_3	Rejection cost	\$10	\$5	\$1	\$5
p	Nonconformance rate	0.05	0.2	0.3	0.01
α	Type I error rate	0.05	0.1	0.05	0.05
β	Type II error rate	0.1	0.05	0.4	0.1
Optimal inspections		0	1	4	2
Savings per piece		n/a	\$1.81	\$23.11	\$56.08
Optimized outgoing defect rate		0.05	0.0137	0.01329	0.000112
k_1/k_2 versus p		$0.33 > 0.05$	$0.05 < 0.2$	$0.01 < 0.3$	$0.00017 < 0.01$
Inspect? (yes/no)		No	Yes	Yes	Yes

Table 4. Example applications and immediate benefits of optimal multiple redundancy model. (Savings per piece indicates difference between optimal and zero inspection.)

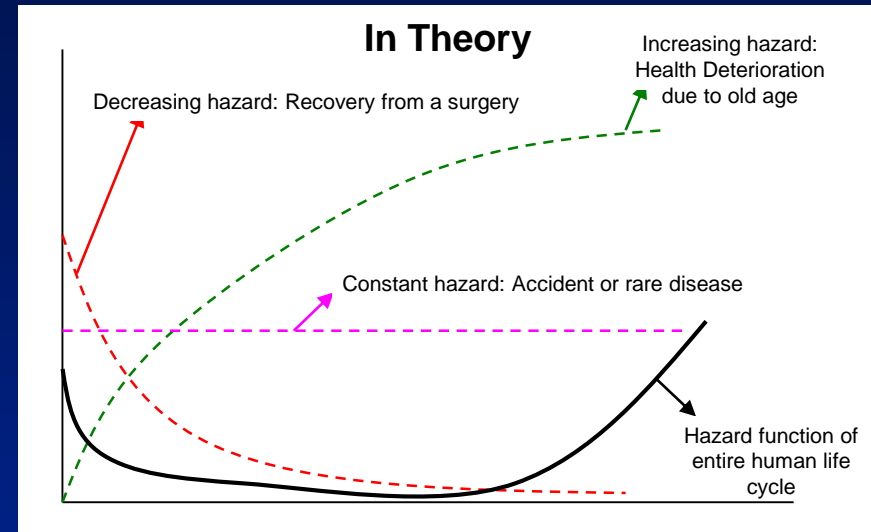
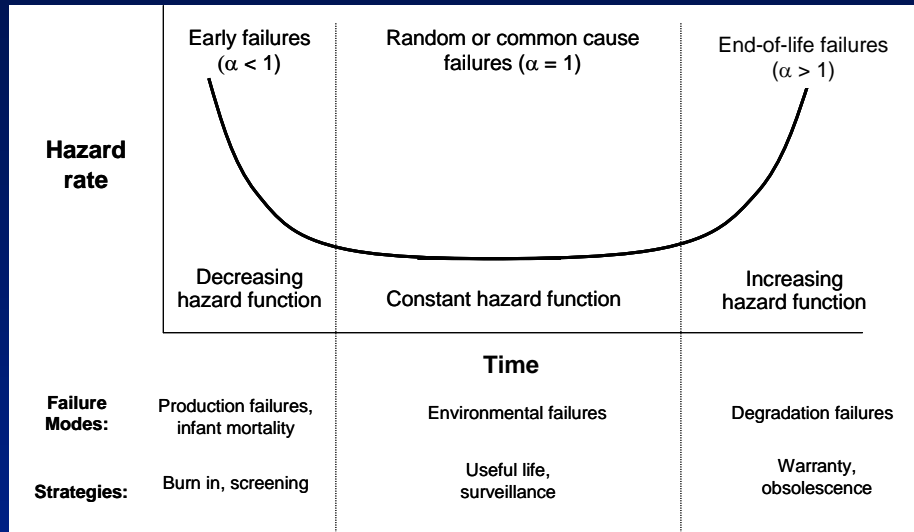
k of n (and failover) reliability models

“Bundle effect”

- If at least k of n measures are met for a patient, the likelihood of adverse outcome significantly decreases
- Non-linear “bundle effect”



3. Reliability bathtub curves



$$h(t) = P(t < T < t + dt) = \frac{f(t)}{R(t)} = -\frac{d}{dt}[\log R(t)]$$

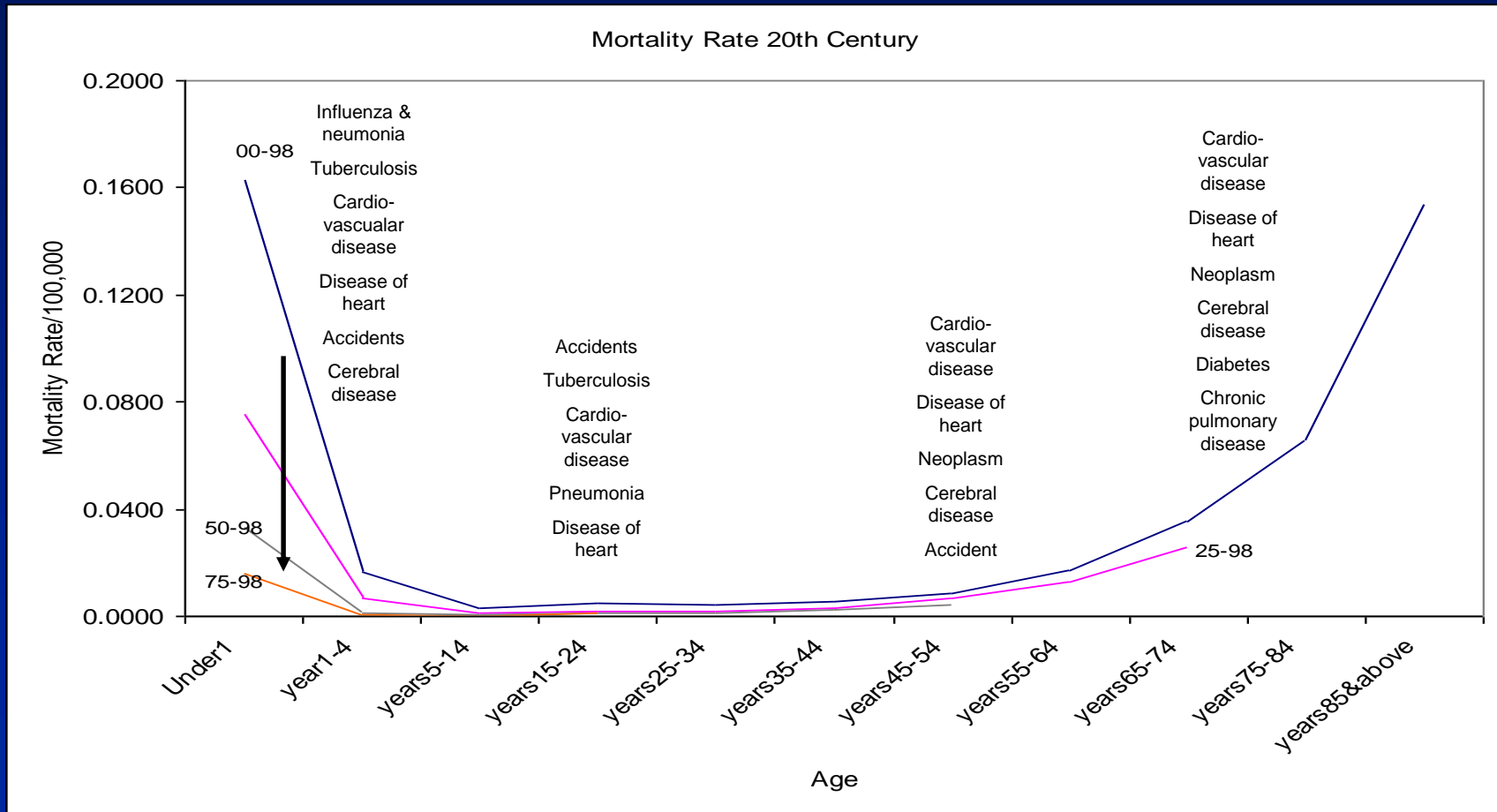
- Different causes and strategies for handling each phase
- Different shapes for different processes

Potential Applications

- Mortality
- Heart failure and other chronic illnesses
- Hospital readmissions
- Returns to OR
- Cancer recurrence

Mortality hazard function

Mortality hazards by year of birth: 1900, 1925, 1950, 1975
(Top reasons for death by age also are listed in rank order, 1900)

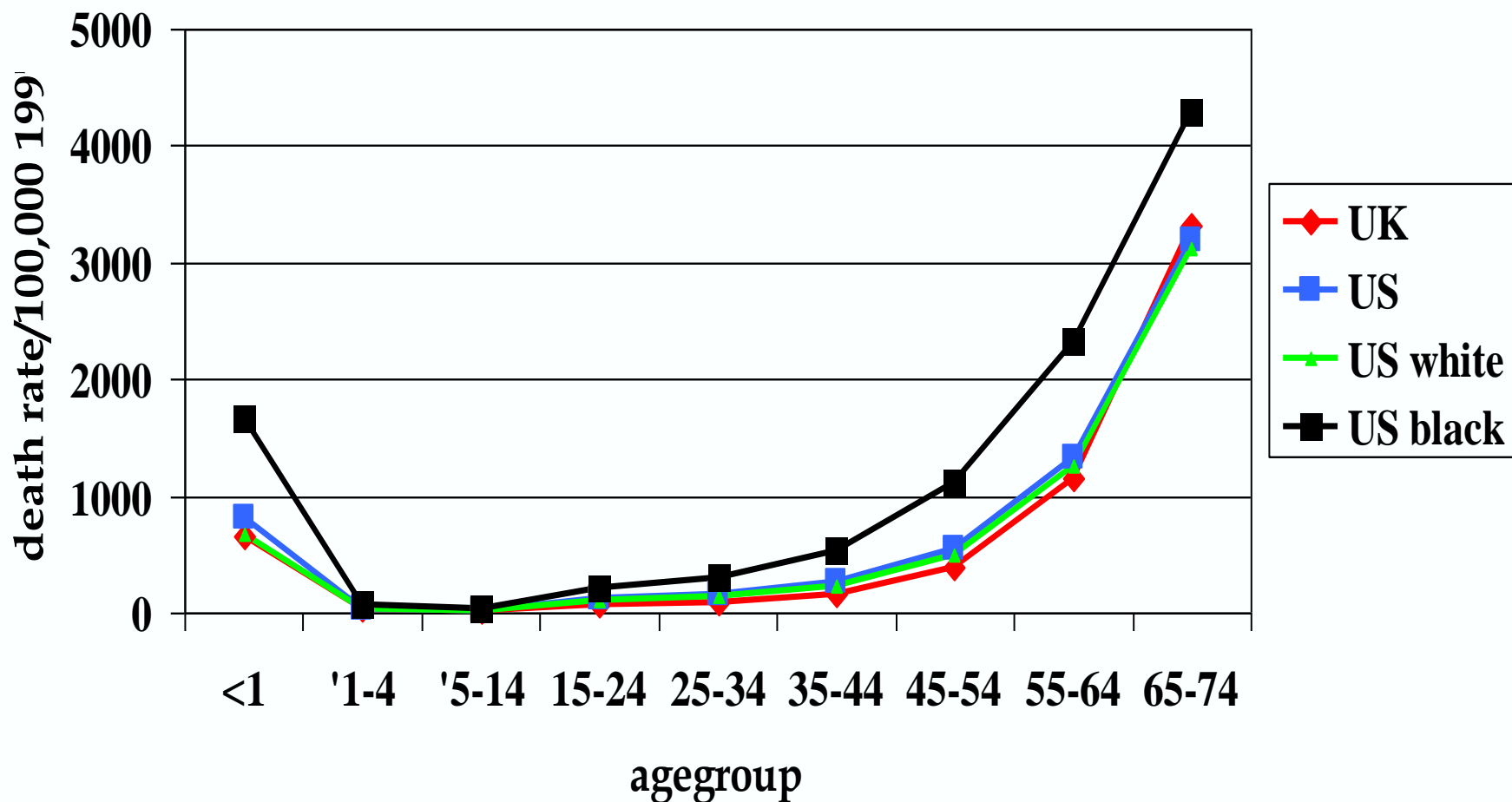


Root cause analysis

Top Reasons for Infant Mortality

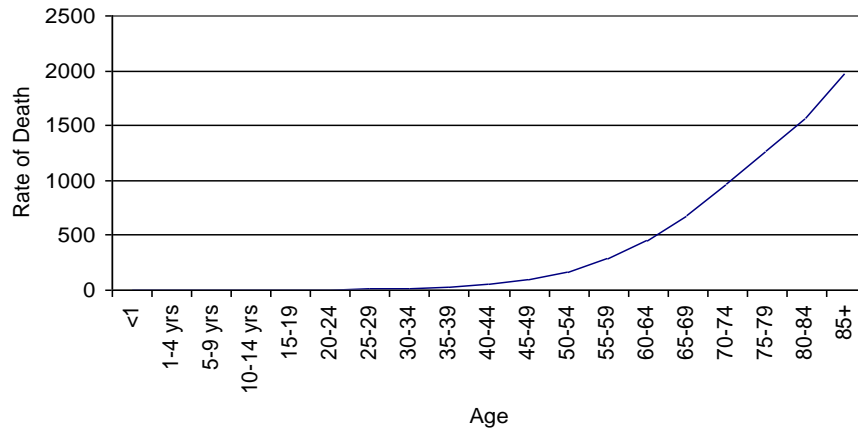
1900	1925	1950	1975	1998
Influenza & pneumonia	Influenza & Pneumonia	Prenatal conditions	Birth injury and labor difficulties	Birth defects
Tuberculosis	Tuberculosis	Congenital anomalies	Congenital anomalies	Pre-term or low birth weight
Cardiovascular disease	Accidents	Pneumonia & influenza	Symptoms and ill defined conditions	Sudden infant death syndrome
Disease of heart	Cardiovascular system	Accidents and adverse effects	Pneumonia and influenza	Pregnancy complications
Accidents	Disease of heart	Disease of heart	Accidents	Respiratory distress syndrome
Cerebral disease	Cerebro vascular disease			Labor difficulties
				Infections
				Accidents
				Pneumonia or influenza

US and UK male death rates, 1997, for whole country (in and out of hospital) vs agegroup

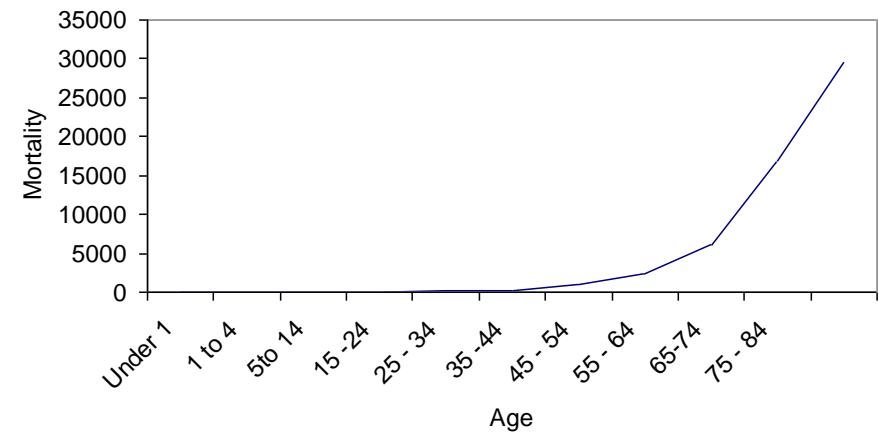


Disease hazard functions

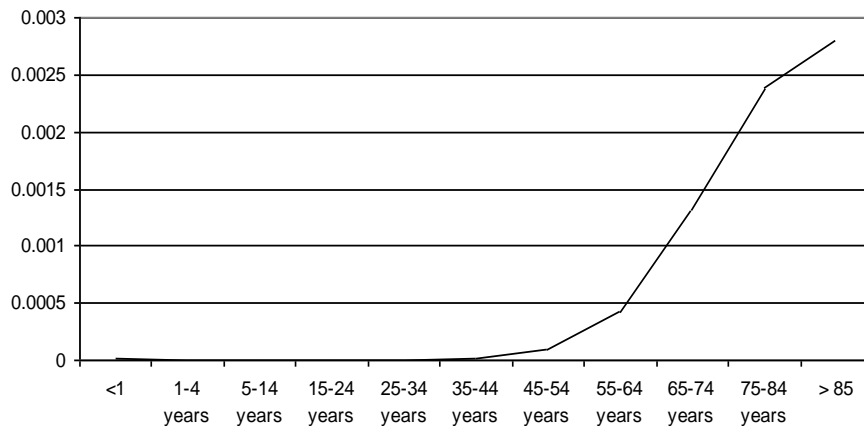
Cancer Mortality 2001



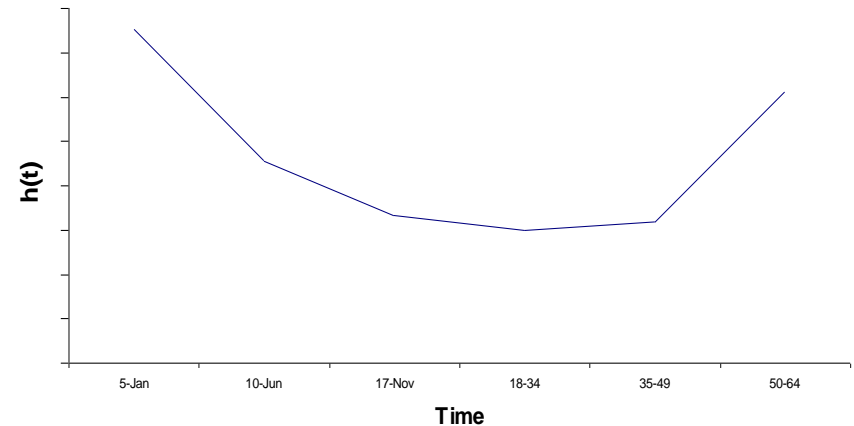
Heart Failure Mortality Rate



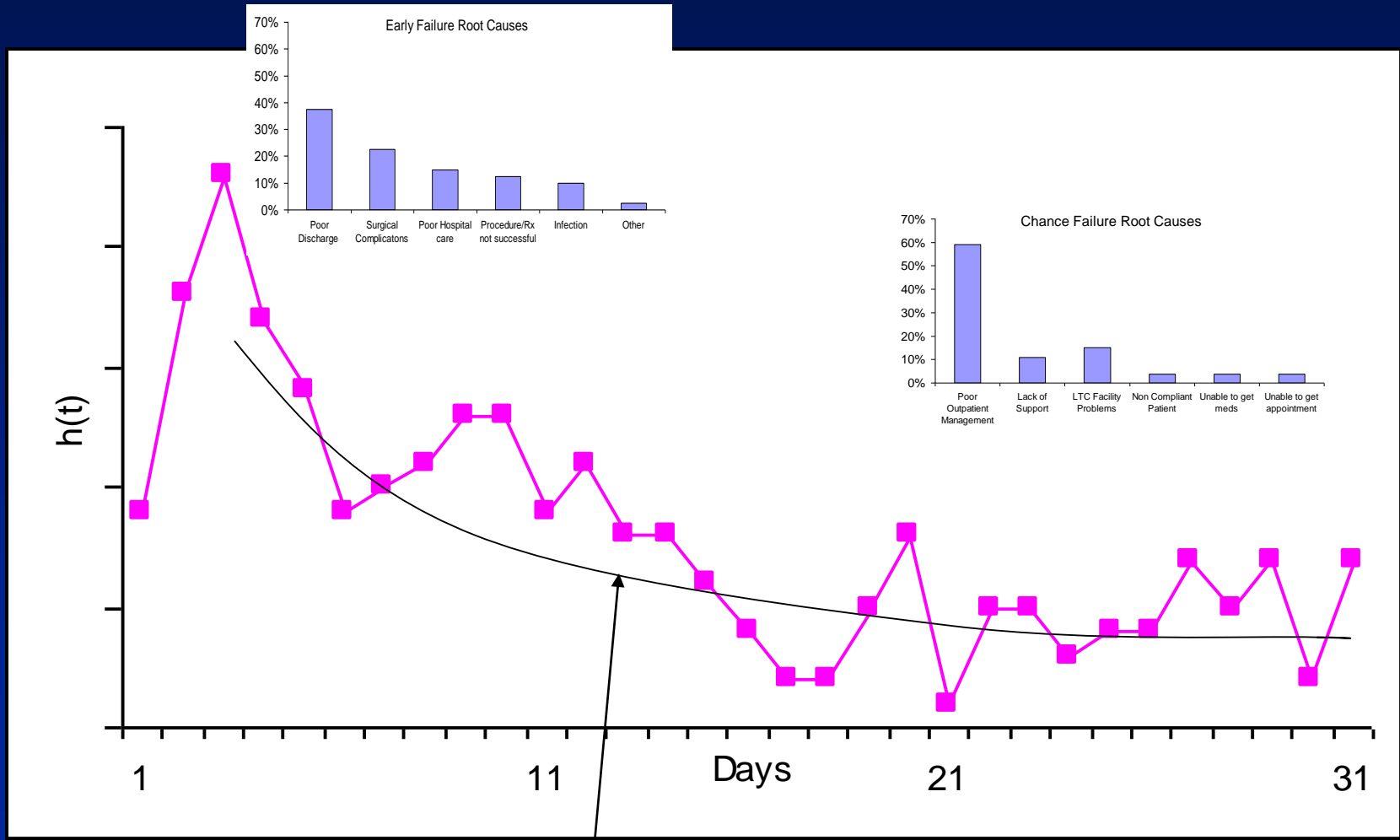
Hazard rate for chronic respiratory illness



Intestine transplant failure



Process hazard functions

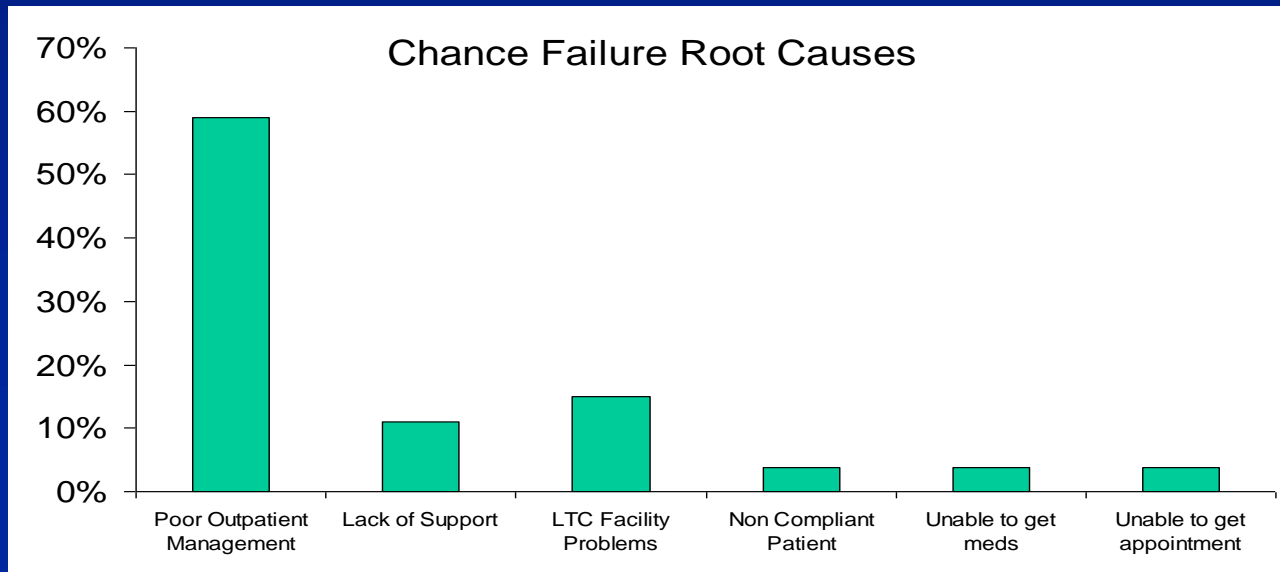
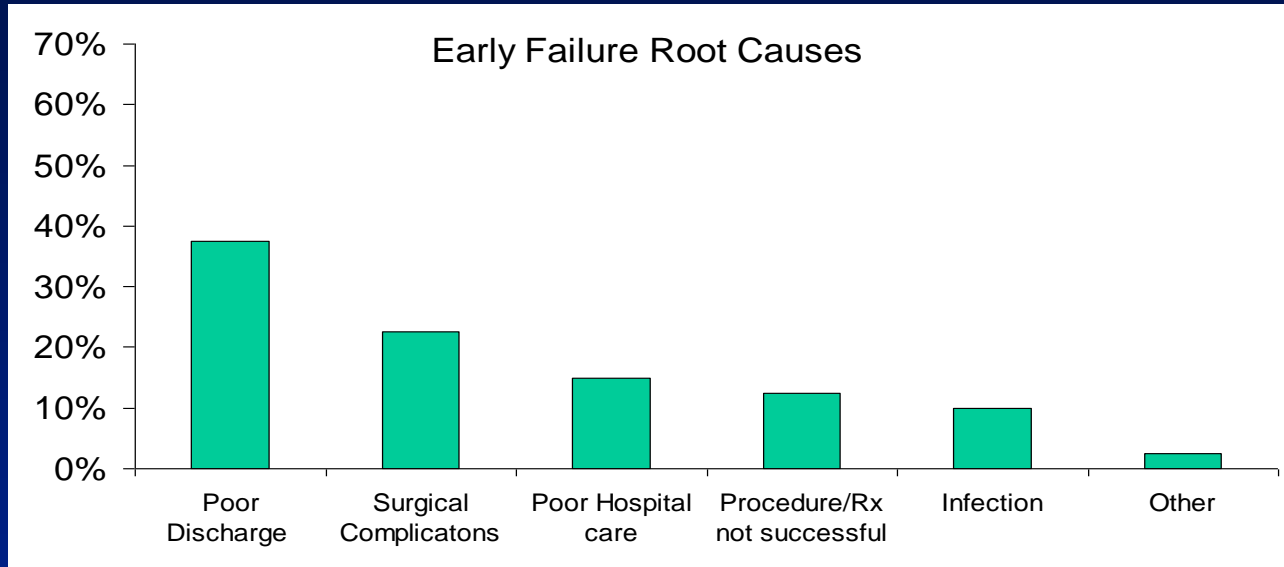


Data follow left-hand side of bathtub curve

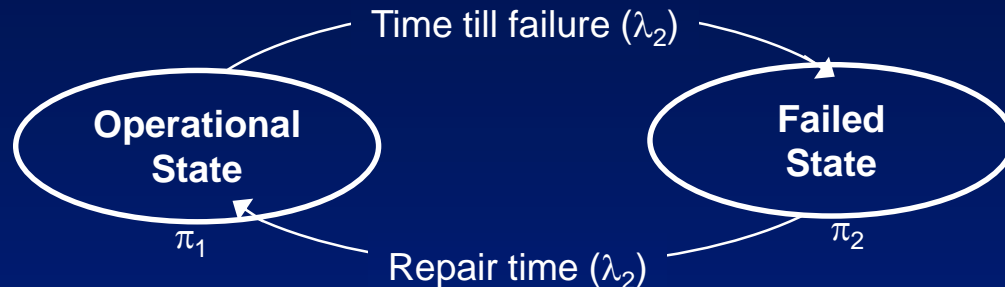
Decreasing failure (readmit) rate as patient time from discharge increases

Failure modes, differing over time

What to focus on when



4. Availability and maintenance models



Human health

- **Patient = complex system**
- **Failure = Degeneration from healthy to sick state**
- **Repair time = Treatment and cure time**
- **Availability = Proportion time in healthy state**

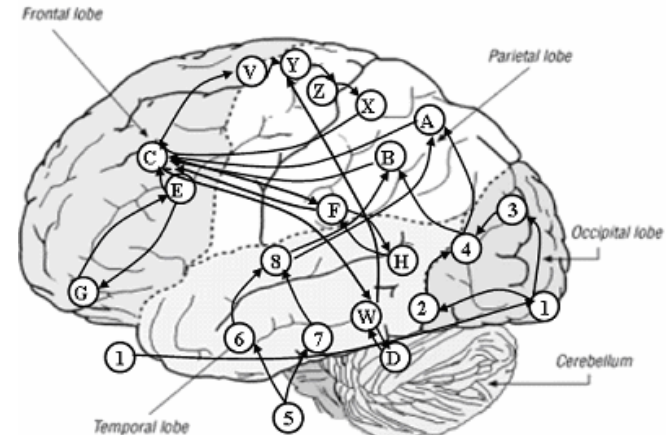
Emergency Dept. Diversion

- **ED = complex system**
- **Failure = ED deteriorates to point of closing**
- **Repair time = Time to get off of diversion**
- **Availability = Proportion time ED open to ambulances**

6. Human factors engineering

Mental load & human performance (queuing network model)

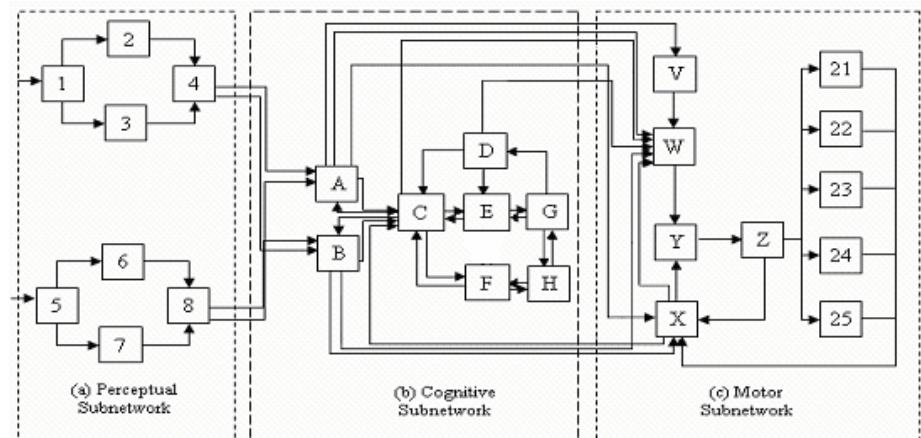
Driving/pilot simulator (NASA)



$$PD = Aa\left(\int_0^T \frac{\lambda_m}{\sum_{j=1}^n \mu_{0,j}} dt\right) / T + b$$

$$TD = EF = PE = FR = Aa\left(\int_0^T \frac{\lambda_{all i}}{\sum_{j=1}^n \mu_{0,j}} dt\right) / 4T + b$$

$$MD = Aa\left(\int_0^T \frac{\lambda_i - \nu_{p,ap,c}}{\sum_{j=1}^n \mu_{0,j}} dt\right) / 3T + b$$



Done!

